REGULAR CONFORMATIONS AND REGULAR STICK NUMBERS OF KNOTS AND LINKS

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Abstract

Let \( K \) be a topological knot or link, and fix \( \alpha \in (0, \pi) \). An \( \alpha \)-regular conformation of \( K \) is a polygonal embedding of \( K \) in \( \mathbb{R}^3 \) with the properties that all edges have the same length and that adjacent edges meet at angle of \( \alpha \). The \( \alpha \)-regular stick number of \( K \), \( S_\alpha(K) \), is the minimal number of edges required to construct an \( \alpha \)-regular conformation of \( K \). There is a lower bound formula for \( S_\alpha(K) \) in terms of the angle \( \alpha \) and the bridge index of \( K \). Generally, this lower bound is not sharp. However, we provide two examples of knots with \( \alpha = \cos^{-1}(-\frac{1}{3}) \) for which this lower bound is achieved. Specifically, we prove that the \( \alpha \)-regular stick number of the trefoil knot, \( 3_1 \), is 11 and the \( \alpha \)-regular stick number of the granny knot, \( 3_1\#3_1 \), is 16. We then construct \( \alpha \)-regular conformations of a certain class of torus links and discuss our work on the calculation of the \( \alpha \)-regular stick numbers of these links.

Biographical Sketch: Timothy Comar is an Associate Professor of Mathematics at Benedictine University. He received his Ph.D. in Mathematics from the University of Michigan in 1996 and completed his undergraduate degree in mathematics at Brown University. Comar has held visiting positions at Saint Louis University and Valparaiso University and has worked at the publisher Addison-Wesley and as a pension actuary at PriceWaterhouseCoopers. Comar was trained as a low-dimensional topologist and did his dissertation work in hyperbolic manifold knots and Kleinian groups. His current research interests have bifurcated between knot theory and mathematical biology. Comar also has a deep interest in undergraduate education. His credits include the development of two semester course sequence for future elementary school teachers and the development of the biocalculus course sequence. He is currently leading a team of mathematicians and biologists who are developing a new textbook and computer laboratory manual for the biocalculus course sequence. Comar is the Principal Investigator on the NSF CCLI grant #DUE-0633232 entitled “Biocalculus: Text Development, Dialog, and Assessment” awarded to Benedictine University and College of DuPage, which will support this writing project, establish biocalculus courses at College of DuPage, and assess the biocalculus courses at these two institutions. Comar is a member of the Education Committee of the Society of Mathematical Biology and is active in the Mathematical and Computational Biology Special Interest Group (BIOSIGMAA) of the Mathematical Association of America.