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Teaching and Service Statement

Teaching becomes more and more connected to research, as we mature and elevate our crafting skills towards designing mathematics, and being able to isolate easy “localized” projects to be completed by students, while clarifying our own ideas. Regarding the service component of our trade, it unfolds naturally, as much our time allows, besides completing our required, pre-determined obligations.

We need to be flexible ...

After many years of practice, and preparing most courses more than once, teaching a course is an opportunity for reflection on the material, sharing with the students not only the mandatory material as specified by the curriculum, but also a view on how the topics fit within mathematics and applications to sciences.

Teaching is an art of improvisation once the lesson plans have been settled after several revisions. This is what gives the student the feeling that mathematics is so interesting, instilling the wish to do well in front of their teacher.

I am used to navigate from an elementary Calculus I course for freshmen, to a historical journey through time and places, in a History of Mathematics course, to focus maybe the next hour on adelic Fourier transform and exact formulas around the Riemann Hypothesis in an advanced topics course. To talk about primes and Riemann zeros, it does not have to be an advanced course; a number theory course, or complex analysis for example, provide opportunities for short “digressions” into territories which can be appreciated, yet without the full details of a rigorous approach.

Regarding strategies, in a big lecture hall with many students, interactions can be generic: a brief review to start with, engages the students and gives them satisfaction, as a bonus for being there, attending. And it is Friday, on a rainy day, with a few attending only, an invitation to huddle together in the front rows may change the initial plan, and go by examples, Moore method, inviting to guess the next step or concept. Although these are exceptions, being able to sense the danger of monotony and routine creeping in and making the student pretend he/she is present, while really checking the emails discretely, is a plus.

... and creative

Mathematics has to be delivered in a modern way, top-down starting from concepts and relations between them, hiding the details as needed, unless the proof is on focus that day. Inventing graphical representations, suggestive enough to help the student connect and relate the abstract, or just difficult concepts, should be a priority. This is where a skilled and seasoned lecturer makes the difference, since otherwise it may seem as just another “download” from the textbook; then why bother coming to class?
Keeping a critical eye on what we teach motivates restructuring the curriculum, creating new courses, and phasing out old ones. Mathematics may not be as dynamic as other sciences, but the curriculum is sometimes so hard to change, it almost stagnates. Then there is the need to just let go of the past, and move on; usually it takes new faculty to initiate the process, faculty with a lot of experience, yet in permanent search of new knowledge: life-long-learners. How we “detect” them? We look at the mathematical areas they have traveled through the years; and maybe changed careers, with confidence and no fear of jumping head first in cold, deep waters …

Another sign of maturity and that they view the “big picture” they can present to the students, is the style of articles they write: sometimes what seem to be essays, organizing a “mathematical district”, establishing links and analogies, connecting to the historical development, may be more important than listing a few more new true statements. The habit of generalizations and “publish of perish” have done enough damage to mathematics, when weeds growing took over the initial goals: to develop beautiful theories, most of the time related to physics.

But this was again about research. But it does not have to be advanced research; an inquisitive mind can research and yield better content to teach at all levels of mathematics, elementary and advanced.

What is really needed is creativity; it then applies to research, teaching and service alike.

Need for a “new math”

Returning to the way mathematics needs to be designed, written and delivered, we need to distinguish between the interface and its implementation. Here teaching may take a part of the load, consisting in a dynamical update of the form and content: redesigning mathematics, even if only through a set of online notes, or just on paper, distilling the traditional expositions into a “user friendly” form.

At the lower levels, the access to powerful computer mathematical packages (SAGE, GAP etc.) and software (Mathematica, Maple etc), can speed up the teaching pace. Let computers do what they do best, and let the students reason, formally and qualitatively; and have them make mistakes, and then recover, not just prepare good followers.

On the other hand, we have to help students from general education courses to develop solid computational skills. So, it depends on the type of course; that’s where the discerning capabilities of a faculty pays-off, and just an algorithmically developed curriculum can do only so much.

Again, many things can be said to describe one’s view on teaching, and especially to improve it, but nothing beats a good example …
Example: Variations on a Theme by Euclid

The “Plot”

This example shows how some standard material of number theory may be creatively enhanced conceptually, related to physics using analogy as a powerful pedagogical anchoring tool, and dressed in graphical “cloths”, to appeal to several “senses” of the students.

The “Departure”

Euclid’s proof that the prime numbers are infinite is a classical example of simplicity, and of his mathematical genius. It relies on the apparently ad-hoc “trick” $N=p_1 \cdots p_n + 1$, where $p_i$ are all the primes (assumed finitely many), which turns out to be a way of building “new primes from old” (common theme throughout curriculum, e.g. Calculus).

Call for “Adventure”

But what if we view this as a physics process of “alchemy”? (For more details see the Research Statement). The “input” primes, as elementary “atoms” (indivisible), interact (fusion), and the resulting products, after “fission”, are new “atoms”.

In an actual class of History of Mathematics, while playing with the trick, the students suggested to start with random primes, then obtaining “other” primes then those entering the process.

The “Surprise”

Then, why not reversing the process? This is an opportunity for the teacher to emphasize the importance of “Inverse Problems” (from solving equations to inverse Galois Theory). What results is precisely the factorization of $q-1$, and if repeated, results in the Pratt tree of the given “output” prime $q$ (see The Research Statement for additional details).

The “Home run”

Then the “big picture” of the partial order of primes emerges (POSet), forming an infinite graph with its source, the “odd” even prime 2 (like the Nile etc.). The reaches of this new structure, at students reach through their simplicity, form a nice departure from the standard classical, “old” paths usually taken within an elementary Number Theory course. Parts of this topic are also suited for K-12 curriculum.
Regarding The Service Component

I will mention a few non-standard activities which I believe fall within this category, aside for the “trivial” ones (participating in committees, meetings, reviewing articles etc.) and the other related in a more direct to teaching and research (curriculum development, writing textbooks, organizing seminars etc.).

Professional networks. Connecting with colleagues electronically, and informally, outside of the direct intra-departmental interactions, is a way to build a network, and belong to the world at large of mathematicians and professionals. With the event of the web, this is easy and, while it may seem a bit “waste of time, the analog of being friends on Facebook, or connected on LinkedIn and similar web networks.

Thank you notes. We often write short notes for our courses, outlines which we make public, and quite often read, use some colleagues’. But rarely we stop and give a “like” to such products, since they usually are not posted on Facebook … So, an occasional question or comment sent to the author, with a thank you note or congratulation, as a follow up, can go a long way in rewarding us, in lack of, or while waiting for, a cash price or official award from the establishment.